

Seismic Design Methodology for Wall Systems

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ABSTRACT

The current understanding of the force, stiffness and yield displacement relationship for structural elements is based on a particular set of assumptions which is widely accepted and rarely questioned. However, there exist certain elements where these assumptions do not apply. (Priestley, 1993 and Paulay, 1997) Flexural walls are one family of elements where a different strength-stiffness-yield displacement relationship exists. It is the intention of this paper to begin with a brief explanation of a more appropriate set of rules for the behaviour of walls and to compare this improved relationship with the traditional understanding. Moreover, this paper will explore the implications of this relationship as it pertains to the design of these elements for both elastically designed systems and inelastic systems. The influence of the load reduction factor, and its relationship with element ductility, is investigated and important conclusions are gathered.

INTRODUCTION

The traditional approach to obtain the seismic base shear of a structure starts with the assumption that the lateral stiffness, or equivalently the period of the structure can be estimated first. Then, the elastic base shear is obtained from the elastic design spectrum. To arrive at the design base shear, the elastic base shear is modified by a reduction factor R . In this approach, it is implicitly assumed that the stiffness of the structure will not be affected when the design strength is reduced from the elastic strength. Using the elasto-plastic representation as a first approximation, structures designed using different R values would have force-displacement relations as shown in Fig.(1a). Structural systems having elastic stiffness independent of yield strength will be referred to as traditional structural systems. Recently it has been shown (Priestley 1993, Paulay 1997, and Priestley and Kowalsky 1998) that ductile reinforced concrete walls have different characteristics. The yield curvature of a wall with a given length is approximately the same, irrespective of the amount of longitudinal reinforcement used. This implies that cantilever flexural wall elements having the same length but having different strengths have force-displacement relations as shown in Fig. (1b). The wall stiffness is proportional to its yield strength, with the constant of proportionality being the yield displacement of the wall.

For structures using structural walls as lateral load resisting elements, it is no longer appropriate to determine the stiffness, period and then the elastic base shear of such structures in sequence. Since the strength and stiffness are dependent, they need to be determined simultaneously in the design process. The object of this paper is to examine a seismic design methodology that is appropriate for structural flexural wall systems.

DETERMINATION OF ELASTIC BASE SHEAR

To make the design methodology as transparent as possible, a single mass system supported by a set of equal length walls will be chosen as the structural model. This single degree of freedom (SDOF) structural model consists of a top mass, $m=1296$ Mg, supported by three equal massless walls of length $l_w=3$ m, thickness $b_w=0.4$ m, and height, $h_w=15$ m. It is assumed that the walls will be detailed such that shear and buckling failures will not occur, and that shear deformation and $P-\Delta$ effects are negligible. Therefore, only flexural response of the walls is considered.

Approximating the moment-curvature relationship as elasto-plastic, the yield curvature can be computed using the formula (Paulay 1997)

$$\phi_y = \frac{1.56f_y}{E_s I_w} \quad (1)$$

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The force-displacement relations can be derived using principles of structural mechanics. The relationship remains elasto-plastic with the yield displacement Δ_y given by

$$\Delta_y = \phi_y \left(\frac{h_w^2}{3} \right) \quad (2)$$

Taking the yield strain of steel to be 0.2%, the yield displacement of the individual wall, and also of the wall system as a whole is equal to 0.078 m. The yield strength F_y and the stiffness K of the wall system must satisfy the relation

$$F_y = \Delta_y K = 0.078K \quad (3)$$

The design methodology is illustrated by designing such a structural model to the Uniform Building Code (UBC-1997) seismic zone 3 rock site requirement. For simplicity, it is assumed that the period of the model lies within the constant spectral velocity region of the UBC spectrum. Modification of the methodology for systems with shorter periods has been presented by Smith (Smith, 1998). There are two approaches to arrive at the elastic base shear of the structure, namely, the iterative approach and the integrated approach.

A) The Iterative Approach

The iterative approach follows the traditional assumption that the system stiffness remains constant regardless of the design strength, and the system stiffness can be estimated based on the dimensions of the walls. Taking the effective moment of inertia of the set of walls $I_{eff} = 2.06 \text{ m}^4$ and using $K = 3EI_{eff}/h_w^3$, the system stiffness (K) is 51176 kN/m and the system period (T) is 1.0 s. Within the period range of interest, the UBC design spectrum for a zone 3 rock site, is given by

$$F_e = mg \frac{C_v}{T} \quad (4)$$

where $C_v = 0.3$ is the seismic coefficient for the site considered. Using the UBC spectrum, the elastic base shear of the wall system $F_e = 3815 \text{ kN}$. However, such a yield strength is not compatible to a system having a yield displacement of 0.078m and a stiffness of 51176 kN/m according to eqn.(3).

An iterative procedure is necessary to arrive at a feasible design where the system strength and stiffness are compatible. Starting with an initial estimation of the system stiffness K_1 , an initial period T_1 is calculated. For this period, the strength demand F_1 is determined using the design spectrum. The stiffness that is compatible with this value of design strength is $K_2 = F_1/\Delta_y$, where Δ_y is the known yield displacement of the system. The stiffness approximation is then revised and the calculations are repeated. After several iterations, convergence on the elastic period (T_e) and the elastic base shear (F_e) will occur. Table (1) gives some sample calculations using this iterative procedure to obtain the elastic strength of the structural model. The design parameters eventually converge to an elastic base shear $F_e = 3645 \text{ kN}$ with the period $T_e = 1.05 \text{ s}$ and stiffness $K_e = 46738 \text{ kN/m}$.

B) The Integrated Approach

The characteristics of the 3 m long wall system shown in eqn.(3) implies that the design yield strength is proportional to the system stiffness. Schematically, the equation can be represented by a straight line in the yield strength-stiffness space, with a slope equal to Δ_y . Replacing the system stiffness K by the system period T , the characteristics of the 3 m wall system can be re-written as

$$F_y = \frac{4\pi^2 m \Delta_y}{T^2} \quad (5)$$

A plot of eqn.(5) in the yield strength-period space gives the locus of feasible design solutions for the 3 m wall system. Superimposing the UBC design spectrum on the same plot, the intersection of the two curves will simultaneously give the elastic base shear F_e , and the system period, T_e as shown in Figure 2.

The intersection of the curves can also be obtained analytically. Since the design is based on the elastic spectrum, the design strength is the yield strength. Equating equations (4) and (5), the elastic period T_e is obtained as

$$T_e = \frac{4\pi^2 \Delta_y}{gC_v} \quad (6)$$

Substituting the yield displacement of the wall system into equation (6) leads to the elastic period $T_e = 1.05$ s. Once T_e is known, an elastic base shear of 3645 kN can be determined using the UBC design spectrum. The advantage of using the integrated approach is that the elastic base shear can be obtained directly and no iteration is necessary.

DETERMINATION OF DESIGN BASE SHEAR

The design base shear of most structural systems in seismic regions is modified from the elastic base shear by a reduction factor R . For a wall system where the system yield strength and stiffness are coupled parameters, a reduction in design strength would lead to a corresponding reduction in stiffness. This would in turn elongate the period of the system. In the descending portion of the UBC spectrum, this period elongation reduces the expected demand on the strength reduced system. The actual demand to design base shear ratio will be less than R . This relationship is graphically illustrated in Figure 3a where the subscript "e" denotes the design parameters associated with a system having a strength equal to the elastic base shear and the superscript "*" denotes parameters associated with the strength reduced system.

The period of the strength reduced system T^* can be determined by expressing T^* and the elastic period T_e in terms of the corresponding system stiffness K^* and K_e respectively. This in turn leads to a relation between T^* and T_e given by

$$\frac{T^*}{T_e} = \sqrt{\frac{K_e}{K^*}} = R^{1/2} \quad (7)$$

Knowing the period elongation, the relationship between the period adjusted demand and the design strength can be obtained algebraically. The design base shear (F^*) corresponding to a load reduction factor of R , is

$$F^* = \frac{F_e}{R} = \frac{mgC_v}{RT_e} \quad (8)$$

This is represented by point b in Figure 3a. The elastic demand for a system with period T^* , represented by point c in the figure, is given by

$$F(T^*) = mg \frac{C_v}{T^*} \quad (9)$$

Using equations (7-9), the effective reduction factor (R_{eff}) is found to be

$$R_{eff} = \frac{F(T^*)}{F^*} = R^{1/2} \quad (10)$$

As an example, consider the SDOF wall model described previously having an elastic base shear $F_e=3645$ kN, a corresponding period $T_e=1.05$ s. and a system stiffness $K_e=46738$ kN/m. Using a load reduction factor $R = 4$, the system design strength $F^*=911$ kN and the system stiffness K^* is also reduced to $K^*=K_e/4=11684$ kN/m. The corresponding period T^* is elongated to $(\sqrt{4})(1.05)$ or 2.10 seconds and the elastic demand for a system with such a period is 1825 kN. Therefore, the elastic demand to actual design strength ratio is $R_{eff}=1825/911 \approx 2$. This is consistent with the prediction using eqn.(10).

A second important consequence of the period elongation is the change in seismic displacement. For the period range considered, the maximum displacement of the strength reduced system is approximately equal to the maximum elastic displacement of a system with the same period T^* . The reduced strength system will therefore have a larger displacement than the system designed with elastic period T_e . This relationship is represented in Figure 3b where the UBC displacement spectrum is shown. The seismic displacement of the system with period T_e , (Δ_e) and that of the strength reduced system (Δ^*) can be written as

$$\Delta_e = \frac{gC_v T_e}{4\pi^2} \quad \text{and} \quad \Delta^* = \frac{gC_v T^*}{4\pi^2} \quad (11)$$

Therefore, the seismic displacement Δ^* of the strength reduced system is given by

$$\Delta^* = \frac{T^*}{T_e} \Delta_e = R^{1.2} \Delta_e \quad (12)$$

Since Δ_e represents the displacement of a system designed with the elastic base shear, it is also the yield displacement Δ_y of the walls. Therefore, the expected ductility of the system is

$$\mu = \frac{\Delta^*}{\Delta_y} = \frac{\Delta^*}{\Delta_e} = R^{1.2} = R_{eff} \quad (13)$$

This means that Newmark's equal displacement hypothesis (Valetsos and Newmark, 1960) is also applicable to wall type systems if one allows for the period elongation effect of the strength reduced system and equates the ductility demand with R_{eff} instead of the nominal reduction factor R . The ductility demand in a strength reduced wall system is not the result of the reduction of the yield displacement, but the result of an increase in seismic displacement associated with the system.

To confirm this theoretical derivation, a family of SDOF wall systems was designed based on the UBC zone 3 rock site spectrum using load reduction factors (R) between 1 and 6. The load-deflection curves of these models were taken to be elasto-plastic. Four original earthquake time history records were used as seeds for the program SYNTH (Naumoski, 1985) to generate four spectrum compatible records as excitation inputs. A 5% viscous damping is specified in the computation to obtain the responses of the SDOF systems. The results of the analysis are presented in Figure 4. The mean ductility demands obtained for the four spectrum compatible records are plotted against the reduction factor R . The dispersion of the response is represented by the bar charts to show the maximum and minimum ductility demands. Shown in the same plot is a curve representing the relationship $\mu=R_{eff}=R^{1.2}$. It is seen that this R_{eff} curve predicts the mean ductility demands well for $R \leq 4$, but overestimates the mean demands for system with R values higher than 4.

CONCLUSIONS

The following conclusions are drawn from this study:

1. Because the yield strength and stiffness of reinforced concrete walls are related, the traditional steps used in obtaining the elastic base shear need to be modified when applied to structures consisting of structural walls. An integrated method which incorporates the yield strength-stiffness characteristics of wall elements is outlined to obtain the elastic base shear for such structures.

2. A reduction from the elastic base shear to obtain the design base shear of wall systems implies that there will be a period elongation of the strength reduced wall system. There are two consequences of this period elongation. First, the elastic strength demand is reduced and second, the seismic displacement will be increased.

3. The ductility demand-strength reduction factor relation for wall structures is given by $\mu = R^{1/2}$. Therefore, using the traditionally accepted relation $\mu = R$ to estimate the ductility demand of wall structures can be very conservative.

4. The seismic displacement of the strength reduced wall system is $R^{1/2}$ times the elastic displacement based on design period of T_e . Using the traditional notion that these two displacements are approximately equal can result in significant under-estimation of the seismic displacement of wall systems.

ACKNOWLEDGMENT

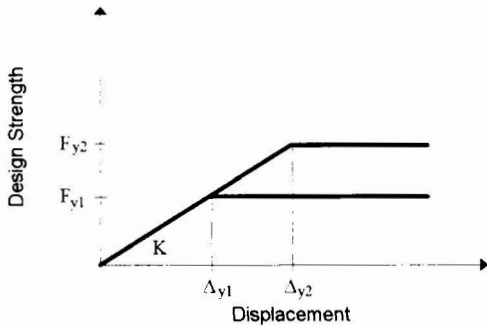
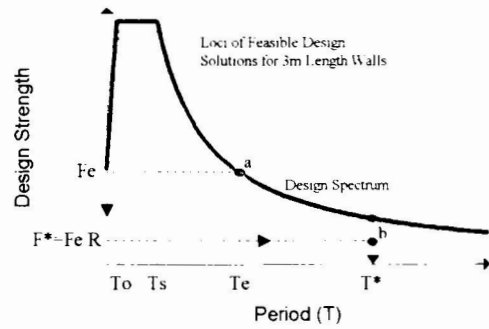
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REFERENCES

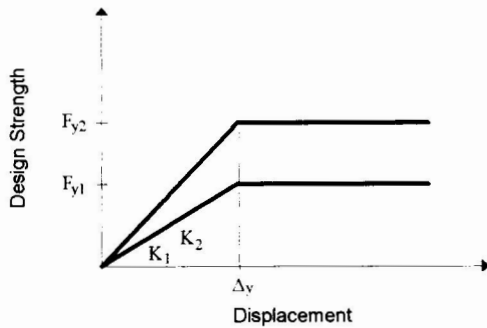
- Naumoski, N. 1985. SYNTH: A program for the generation of artificial accelerograms compatible with a target spectrum. Department of Civil Engineering, McMaster University.
- Paulay, T. 1997. "Seismic Torsional Effects on Ductile Structural Wall Systems", *Journal of Earthquake Engineering*, Vol. 1, No. 4, pp. 721-745
- Priestley, M.J. Nigel, 1993. "Myths and Fallacies in Earthquake Engineering - Conflicts Between Design and Reality". *Bulletin of the New Zealand National Society for Earthquake Engineering*, Vol. 26, No. 3 pp. 329-341. This paper reprinted in *Concrete International*, Vol. 19, No. 2, February 1997. pp. 54-63
- Priestley, M.J.N. and M.J. Kowalsky, 1995. "Aspects of Drift and Ductility Capacity of Rectangular Cantilever Structural Walls", *Bulletin of the New Zealand National Society for Earthquake Engineering*, Vol. 31, No. 2, June 1998. pp. 73-85.
- Smith, R.S.H. 1998. "Design Methodologies for Ductile Flexural Wall Systems". Thesis for the degree of Master of Engineering, McMaster University.
- UBC, 1997. "Uniform Building Code Volume 2: Structural Engineering Design Provisions", International Conference of Building Officials, Whittier, California.
- Valetsos, A.S. and N.M. Newmark, 1960. "Effect of Inelastic Behavior on the Response of Simple Systems to Earthquake Motions", *Proceedings of the Second World Conference on Earthquake Engineering*, Volume 2, Japan.

Table 1. Iterative Approach to Determine Elastic Base Shear.

K (kN/m)	T (sec.)	S _a (g)	F (kN)
51176.4	0.999879	0.300036	3814.589
48904.99	1.022835	0.293302	3728.975
47807.37	1.034511	0.289992	3686.891
47267.84	1.040398	0.288351	3666.028
47000.36	1.043354	0.287534	3655.64
46867.19	1.044836	0.287127	3650.458
46800.74	1.045577	0.286923	3647.869
46767.55	1.045948	0.286821	3646.576
46750.97	1.046133	0.28677	3645.929
46742.68	1.046226	0.286745	3645.606
46738.53	1.046273	0.286732	3645.444



(a) Traditional Structural Element.



(b) Flexural Wall Element.

Figure 1. Force-Displacement Relationship.

(a) Traditional Structural Element.

(b) Flexural Wall Element.

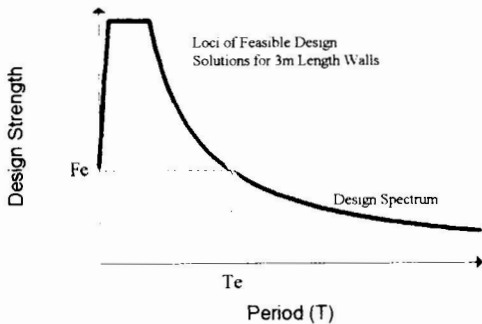
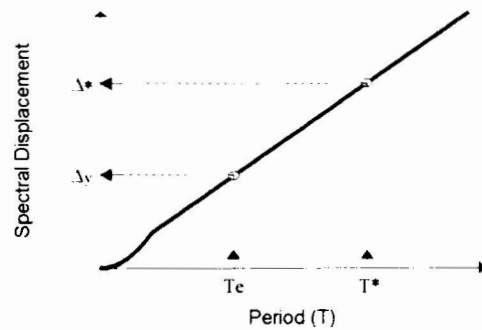


Fig 2. Integrated Approach to Determine Elastic Base Shear.

a) Seismic Demand to Design Strength Ratio.



b) Expected Seismic Displacement.

Fig 3. Effects of Strength Reduced Design On
(a) Seismic Demand to Design Strength Ratio.
(b) Expected Seismic Displacement.

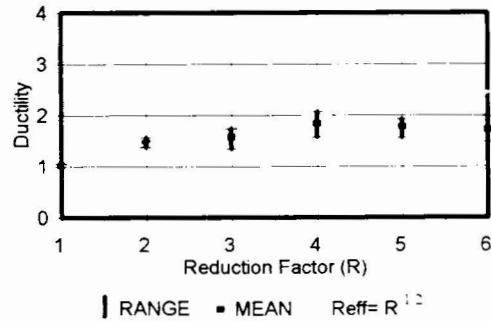


Fig 4. Comparing Actual and Predicted Ductility Demand